CALIBRATION OF POLARIMETRIC RADAR SYSTEMS[1]

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1. INTRODUCTION

The calibration of reciprocal radar systems has been studied in [1], where it was shown: (1) that full polarimetric calibration of radar systems can remove cross-polarization errors from the measurements, and (2) that for *reciprocal* radars Fourier analysis of polarimetric data obtained using a rotating dihedral can reduce noise and background errors in the calibration. To calibrate *nonreciprocal* radar systems [2], we must obtain full polarimetric data from two objects whose scattering matrices have independent eigenvectors [3]. Thus, in addition to a rotating dihedral, a sphere or a flat plate is needed to solve for the transmitting and receiving characteristics of the system [3,4,5]. We find the current methods of solution of the calibration equations unattractive for the following reasons: (1) noise and clutter rejection is not built into the analysis, and (2) the mathematics seems unnecessarily complicated. In addition to mathematical simplification, several ways to verify data integrity are presented, so that system problems can be detected early in the calibration phase.

2. POLARIMETRIC CALIBRATION OF NONRECIPROCAL RADARS

The measured signal $\mathcal{M}$ received from a target is given by

$$\begin{pmatrix} \mathcal{M}_{hh} & \mathcal{M}_{hv} \\ \mathcal{M}_{vh} & \mathcal{M}_{vv} \end{pmatrix} = \mathcal{K} \begin{pmatrix} R_{hh} & R_{hv} \\ R_{vh} & R_{vv} \end{pmatrix} \begin{pmatrix} \mathcal{A}_{hh} & \mathcal{A}_{hv} \\ \mathcal{A}_{vh} & \mathcal{A}_{vv} \end{pmatrix} \begin{pmatrix} T_{hh} & T_{hv} \\ T_{vh} & T_{vv} \end{pmatrix}, \tag{1}$$

where $\mathbf{R}$ and $\mathbf{T}$ are the receiving and transmitting characteristics of the radar system, $\mathcal{A}$ is the scattering matrix of the target, and $\mathcal{K}$ is a complex constant containing phase and distance information. For *nonreciprocal systems* no *a priori* relationship is assumed among the elements of $\mathbf{R}$ and $\mathbf{T}$. To solve for the two unknown matrices $\mathbf{R}$ and $\mathbf{T}$, we need several independent measurements. Let $\mathcal{M}_1$ and $\mathcal{M}_2$ be two such measurements, and $\mathcal{A}_1$ and $\mathcal{A}_2$ be the corresponding *known* calibration target scattering matrices; then we can eliminate either $\mathbf{T}$ or $\mathbf{R}$ from the expression for the received signal. Thus,

$$\mathcal{M}_1\mathcal{M}_2^{-1} = \mathcal{K}_1\mathcal{K}_2^{-1}\mathbf{R}\mathcal{A}_1\mathcal{A}_2^{-1}\mathbf{R}^{-1}, \tag{2}$$

and, similarly,

$$\mathcal{M}_1^{-1}\mathcal{M}_2 = \mathcal{K}_1^{-1}\mathcal{K}_2\mathbf{T}^{-1}\mathcal{A}_1^{-1}\mathcal{A}_2\mathbf{T}. \tag{3}$$

We will solve the nonlinear equations (2) in a simple manner.
The determinants of the above matrices satisfy the relationships

$$|\mathcal{M}| = \mathcal{K}^2|\mathbf{R}||\mathcal{A}||\mathbf{T}|, \tag{4}$$

$$|\mathcal{M}_1\mathcal{M}_2^{-1}| = \mathcal{K}_1^2\mathcal{K}_2^{-2}|\mathcal{A}_1\mathcal{A}_2^{-1}| \tag{5}$$

and

$$|\mathcal{M}_1^{-1}\mathcal{M}_2| = \mathcal{K}_1^{-2}\mathcal{K}_2^2|\mathcal{A}_1^{-1}\mathcal{A}_2|. \tag{6}$$

$|\mathcal{M}|$ determines $|\mathbf{R}||\mathbf{T}|$, and $|\mathcal{M}_1\mathcal{M}_2^{-1}|$ and $|\mathcal{M}_1^{-1}\mathcal{M}_2|$ depend only on the calibration targets, not on the measurement system.

These determinant relationships can be exploited to monitor both data integrity and the correctness of the mathematical model of the calibration target scattering matrix.

3. CALIBRATION USING A DIHEDRAL AND A SPHERE

Let the scattering matrices of a 90° dihedral and of a sphere be given by $\mathcal{D}(\theta)$ and $\mathcal{S}$, where

$$\mathcal{D}(\theta) = d_0 \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}, \tag{7}$$

$$\mathcal{S} = s_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{8}$$

and $|\mathcal{D}(\theta)| = -d_0^2$ independent of θ, and $|\mathcal{S}| = s_0^2$. Let the measurements using the dihedral and the sphere be denoted by $\mathcal{M}_{\mathcal{D}}(\theta)$ and $\mathcal{M}_{\mathcal{S}}$. In general, the dihedral measurements can be be written as

$$\mathcal{M}_{\mathcal{D}}(\theta) = \begin{pmatrix} c_{hh}\cos 2\theta + s_{hh}\sin 2\theta & c_{hv}\cos 2\theta + s_{hv}\sin 2\theta \\ c_{vh}\cos 2\theta + s_{vh}\sin 2\theta & c_{vv}\cos 2\theta + s_{vv}\sin 2\theta \end{pmatrix}, \tag{9}$$

where the Fourier coefficients c and s can be written in terms of the components of $\mathbf{R}$ and $\mathbf{T}$. (See Appendix A for details.)

We can attempt to calibrate the system using only dihedral measurements. Thus,

$$\mathcal{M}_{\mathcal{D}}(0)\mathcal{M}_{\mathcal{D}}(\theta)^{-1} = \mathbf{R}\mathcal{D}(0)\mathcal{D}(\theta)^{-1}\mathbf{R}^{-1}. \tag{10}$$

This equation contains $|\mathbf{R}|$ explicitly, and

$$|\mathcal{M}_{\mathcal{D}}(0)\mathcal{M}_{\mathcal{D}}(\theta)^{-1}| = |\mathcal{D}(0)\mathcal{D}(\theta)^{-1}| = 1. \tag{11}$$

This determinant condition can again be used to check the model and data integrity of scattering measurements using a dihedral.

The components of $\mathcal{M}_{\mathcal{D}}(0)\mathcal{M}_{\mathcal{D}}(\theta)^{-1}$ can be written in terms of Fourier coefficients $\tilde{c}$ and $\tilde{s}$, which can be obtained in terms of the Fourier coefficients c and s (see Appendix A). For convenience we set[1]

$$|\mathbf{R}| \equiv r_{hh}r_{vv} - r_{hv}r_{vh} = 1, \tag{12}$$

and obtain the set of equations

$$\begin{aligned} \tilde{c}_{hh} &= 1, \\ \tilde{s}_{hh} &= r_{hh}r_{vh} + r_{hv}r_{vv}, \\ \tilde{c}_{hv} &= 0, \\ \tilde{s}_{hv} &= -r_{hh}^2 - r_{hv}^2, \\ \tilde{c}_{vh} &= 0, \\ \tilde{s}_{vh} &= r_{vh}^2 + r_{vv}^2, \\ \tilde{c}_{vv} &= 1, \\ \tilde{s}_{vv} &= -r_{hh}r_{vh} - r_{hv}r_{vv}. \end{aligned} \tag{13}$$

Ordinarily 3 equations and the determinant condition would be enough to determine the 4 components of $\mathbf{R}$; however, these equations are not independent, since

$$(r_{hh}r_{vh} + r_{hv}r_{vv})^2 + (r_{hh}r_{vv} - r_{hv}r_{vh})^2 = (r_{hh}^2 + r_{hv}^2)^2 + (r_{vh}^2 + r_{vv}^2)^2. \tag{14}$$

[1] In fact, $|\mathbf{R}| \neq 1$. Once $\mathbf{R}$ and $\mathbf{T}$ are determined within a normalization factor, we can use (4) to determine $|\mathbf{R}||\mathbf{T}|$.

Therefore, an additional independent equation, which is obtained using a sphere (or a flat plate), is needed. We write

$$\mathcal{M}_S \mathcal{M}_{\mathcal{D}}(\theta)^{-1} = \mathbf{R}\mathcal{S}\mathcal{D}(\theta)^{-1}\mathbf{R}^{-1}, \tag{15}$$

and we define the normalization $\mathcal{N}$, a constant independent of θ,

$$\mathcal{N} \equiv \pm\sqrt{-|\mathcal{M}_S \mathcal{M}_{\mathcal{D}}(\theta)^{-1}|} = \frac{s_0}{d_0}\frac{\mathcal{K}_1}{\mathcal{K}_2}. \tag{16}$$

Here we have good estimates of the right side from physical optics and the experimental parameters; the sign is chosen so that the equality holds.

We can eliminate s_0/d_0 from the data by computing the expression $\mathcal{M}_S \mathcal{M}_{\mathcal{D}}(\theta)^{-1}/\mathcal{N}$, whose determinant is 1 [4]. We can write its components in terms of Fourier coefficients as for the dihedral. Thus,

$$\begin{aligned}
\hat{c}_{hh} &= -r_{hh}r_{vv} - r_{hv}r_{vh}, \\
\hat{s}_{hh} &= -r_{hh}r_{vh} + r_{hv}r_{vv}, \\
\hat{c}_{hv} &= 2r_{hh}r_{hv}, \\
\hat{s}_{hv} &= r_{hh}^2 - r_{hv}^2, \\
\hat{c}_{vh} &= -2r_{vv}r_{vh}, \\
\hat{s}_{vh} &= r_{vv}^2 - r_{vh}^2, \\
\hat{c}_{vv} &= r_{hh}r_{vv} + r_{hv}r_{vh}, \\
\hat{s}_{vv} &= r_{hh}r_{vh} - r_{hv}r_{vv}.
\end{aligned} \tag{17}$$

This set of equations can be solved for the components of $\mathbf{R}$, but the lower signal levels obtained from the sphere data might degrade the solution. Therefore, we prefer to work with as many dihedral equations as possible. We choose the expressions for $\tilde{s}_{hh}$, $\tilde{s}_{vh}$, $\hat{s}_{vh}$ and $|\mathbf{R}| = 1$ to obtain a set of independent equations. The solutions are

$$\begin{aligned}
r_{vv} &= \pm\sqrt{\frac{\tilde{s}_{vh} + \hat{s}_{vh}}{2}}, \\
r_{vh} &= \pm\sqrt{\frac{\tilde{s}_{vh} - \hat{s}_{vh}}{2}}, \\
r_{hh} &= \frac{r_{vh}\tilde{s}_{hh} + r_{vv}}{\tilde{s}_{vh}}, \\
r_{hv} &= \frac{r_{vh} - r_{vv}\tilde{s}_{hh}}{\tilde{s}_{vh}}.
\end{aligned} \tag{18}$$

The condition $|\mathbf{R}| = 1$ allows us to resolve the sign ambiguity in r_{vv} and r_{vh}. A similar procedure yields expressions for $\mathbf{T}$. The details will not be repeated.

REFERENCES

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Appendix A

THE FOURIER COEFFICIENTS OF DIHEDRAL DATA

The 8 Fourier coefficients in $\mathcal{M}_\mathcal{D}(\theta)$ can be written in terms of the components of $\mathbf{R}$ and $\mathbf{T}$ by expanding the matrix expression $\mathcal{M}_\mathcal{D}(\theta) = \mathcal{K}_\mathcal{D}\mathbf{R}\mathcal{D}(\theta)\mathbf{T}$. We get

$$
\begin{aligned}
c_{hh} &= -r_{hh}t_{hh} + r_{hv}t_{vh}, \\
s_{hh} &= r_{hh}t_{vh} + r_{hv}t_{hh}, \\
c_{hv} &= -r_{hh}t_{hv} + r_{hv}t_{vv}, \\
s_{hv} &= r_{hh}t_{vv} + r_{hv}t_{hv}, \\
c_{vh} &= -r_{vh}t_{hh} + r_{vv}t_{vh}, \\
s_{vh} &= r_{vh}t_{vh} + r_{vv}t_{hh}, \\
c_{vv} &= -r_{vh}t_{hv} + r_{vv}t_{vv}, \\
s_{vv} &= r_{vh}t_{vv} + r_{vv}t_{hv}.
\end{aligned}
\tag{a1}
$$

We can determine the Fourier coefficients $\tilde{c}$, $\tilde{s}$ of $\mathcal{M}_\mathcal{D}(0)\mathcal{M}_\mathcal{D}(\theta)^{-1}$ in terms of the coefficients c, s of $\mathcal{M}(\theta)$. We get

$$
\begin{aligned}
\tilde{c}_{hh} &= 1, \\
\tilde{s}_{hh} &= c_{hv}s_{vh} - c_{hh}s_{vv}, \\
\tilde{c}_{hv} &= 0, \\
\tilde{s}_{hv} &= c_{hh}s_{hv} - c_{hv}s_{hh}, \\
\tilde{c}_{vh} &= 0, \\
\tilde{s}_{vh} &= c_{vv}s_{vh} - c_{vh}s_{vv}, \\
\tilde{c}_{vv} &= 1, \\
\tilde{s}_{vv} &= c_{vh}s_{hv} - c_{vv}s_{hh}.
\end{aligned}
\tag{a2}
$$

These relationships are consistent with $|\mathcal{M}_\mathcal{D}(0)\mathcal{M}_\mathcal{D}(\theta)^{-1}| = 1$ and $|\mathcal{M}_\mathcal{D}(\theta)|/\mathcal{K}_\mathcal{D}^2 = -1$, which imply the auxiliary relationships

$$
\begin{aligned}
c_{hv}c_{vh} - c_{hh}c_{vv} &= 1, \\
s_{hv}s_{vh} - s_{hh}s_{vv} &= 1, \\
c_{hh}s_{vv} + s_{hh}c_{vv} &= c_{hv}s_{vh} + s_{hv}c_{vh}.
\end{aligned}
\tag{a3}
$$

The structure of these relationships can be understood in terms of the polarization symmetry of the dihedral. Again, we can use these relationships to verify data and model integrity.

Similarly, we can express $\hat{c}$ and $\hat{s}$ in terms of c and s.